

Example III

In the case of Yap, 1987, we demonstrate how the estimation of the underlying mortality function can be fine-tuned so as to take the underlying age-pattern of fertility into account. For graduation of the observed fertility rates (see Table 14, column (5)) the graduating polynomial is

$$p_{\phi}(x) = -1.051956 + 0.1109 x - 0.0031026 x^2 + 0.00026111 x^3$$

with approximate roots, $a_1 = 14.9562$, $a_2 = 50.0354$ and $a_3 = 53.8664$. As already noted, the mean age for $p_{\phi}(x)$ is $\hat{\mu} = 29.6$. We use $p_{\phi}(x)$ to model the fertility function f in (4).

To model the mortality function, we use $q(x; a, b)$ with $\hat{a} = -1.5055$ and $\hat{b} = 0.0757$ which gives a continuous representation of the mortality function (both sexes) in West level 20 (see Hartmann, 1980, pp. 36-55).

Starting the numerical integration in (4) at age $x = 14.9562$, the model proportions of deceased children appear as given in Table A5.

To estimate K in (16), we use (17) for the ages 20, 21, 23, 24 and 25 (we drop age 22 because $\hat{H}_{22} = 0$) and obtain

$$\hat{K} = \frac{\sum_x [\hat{H}_x H_x^s]}{\sum_x [H_x^s]^2} = 0.014024 / 0.013593 = 1.0317.$$

As in the previous examples, the estimated mortality function becomes $\hat{q}(x) = 1.0317 q(x; -1.5055, 0.0757)$ with q given by (10).

Table A5.-Model proportions of deceased children corresponding to estimated fertility for Yap 1987

Age x	Cumulated Fertility at age x	Cumulated child deaths at age x	Proportion of deceased children at age x
14.9562	0.00000	0.00000	
15.5	0.00518	0.00018	0.0347
16.5	0.04015	0.00162	0.0403
17.5	0.10498	0.00456	0.0434
18.5	0.19612	0.00896	0.0457
19.5	0.31019	0.01472	0.0475
15-19	0.13132		0.0423
20.5	0.44395	0.02179	0.0491
21.5	0.59434	0.02995	0.0504
22.5	0.75844	0.03916	0.0516
23.5	0.93349	0.04918	0.0527
24.5	1.11690	0.05998	0.0537
20-24	0.76942		0.0515
25.5	1.30621	0.07138	0.0546
26.5	1.49913	0.08330	0.0556
27.5	1.69354	0.09548	0.0564
28.5	1.88746	0.10789	0.0572
29.5	2.07907	0.12040	0.0579
25-29	1.69308		0.0563

2.5 Estimating a life table for Yap around 1987

The estimation of a life table for Yap is necessarily an approximate endeavor. The only practical solution at hand is to take advantage of the estimates of infant and childhood mortality and infer the remaining part of the survival curve from these estimates. With this approach one could select e.g. a West model life table with the corresponding level of infant and childhood mortality. We abandon this approach because, in our view, it would give an estimated life table with a too high life expectancy. Instead, we make use of the Brass logit life table approach (see e.g. Brass, 1971, pp. 69-110). The application of the Brass logit life table method hinges on the selection of a proper standard life table. We have chosen the life table for Western Samoa around 1981-82 as a standard (see Table 2).

The Brass logit life table method makes use of the approximate relation between survival functions $l(x)$ and $l_s(x)$ that

$$\text{logit } l(x) = \nu + \omega \text{ logit } l_s(x) \quad (19)$$

where $l_s(x)$ is referred to as a standard survival function and ν and ω are parameters. For a given standard survival function, (19) generates a family of related survival functions. From a practical point of view, it is best to work with values of ω that are close to unity. The more ω departs from unity, the more the age-pattern of the generated survival function deviates from empirical patterns. For estimates $\hat{\nu}$ and $\hat{\omega}$, and by letting

$$\text{logit } \hat{l}(x) = \hat{\nu} + \hat{\omega} \text{ logit } l_s(x),$$

the generated survival function is

$$\hat{l}(x) = \frac{1}{1 + \exp(2\hat{\omega}) [(1 - l_s(x)) / l_s(x)]^{\hat{\omega}}}$$

In our application of (19), we let $\omega = 1$ and estimate ν on the basis of the difference between childhood mortality in the standard life table (the life table for Western Samoa 1981-82) and in the estimated mortality function for Yap State 1987 (see bottom of Table 10).

In order to take full advantage of the two childhood survival functions, we have fitted (10) by means of (11) to the two childhood experiences. This gives two continuous representations of the survival functions. These are $l_s(x) = 1 - q(x, -1.5604, 0.0852)$ for the standard survival function and $\hat{l}(x) = 1 - q(x, -1.4888, 0.0760)$. Notice that $l_s(x)$ is a continuous representation of the childhood experience for Western Samoa 1981-82, and that $\hat{l}(x)$ is a continuous representation of childhood mortality for Yap in 1987. To estimate ν , we let

$$\hat{\nu} = (1/5) \sum_{x=1}^5 [\text{logit } \hat{l}(x) - \text{logit } l_s(x)]$$

The estimation of ν is illustrated in Table A6.

Table A6.-Continuous representations of the survival functions for the ages 1,2,3,4 and 5 for Western Samoa 1981-82 and Yap 1987

Age	$l_s(x)$ Samoa	$l(x)$ Yap	logit $l_s(x)$	logit $l(x)$	logit $l(x) - \text{logit } l_s(x)$
0	1.0000	1.0000			
1	0.9577	0.9516	-1.5599	-1.4893	0.0706
2	0.9527	0.9464	-1.5014	-1.4356	0.0658
3	0.9495	0.9432	-1.4670	-1.4049	0.0621
4	0.9471	0.9409	-1.4425	-1.3838	0.0587
5	0.9451	0.9390	-1.4229	-1.3670	0.0559

Mean logit difference: 0.0626

To estimate a life table for Yap, we let

$$\hat{l}_x = 0.0626 + \text{logit } l_s(x)$$

where $l_s(x)$ is the survival function (for both sexes) for Western Samoa during 1981-82. The resulting life table is shown in Table 11.

3.0 ESTIMATING FERTILITY FROM DATA ON BIRTHS DURING THE YEAR BEFORE THE CENSUS

Estimating age-specific fertility rates from returns on the number of children born during the year before the census would be a relatively simple matter if the returns were correct. The only problem that arises is that the women, on average, were half a year younger when they gave birth. The displacement of half a year is immaterial in a small population where the estimated rates have large variances. In populations where the reporting of births (in the census) is affected by recall errors, omissions, misunderstandings between the canvassers and the respondents, etc., the age-specific fertility rates estimated from the census returns will, most likely, give a too low total fertility rate. Furthermore, the age-pattern of the estimated fertility schedule may deviate significantly from the true age-pattern of fertility.

In order to make use of census returns concerning the number of children born during the year before the census, the reporting must be fairly accurate. Given that underreporting is the same in all the age groups, the normalized schedule can be used for estimation of age-specific fertility rates from the reported mean parities in the census. This approach, of course, assumes that fertility and mortality are near stationary. Since there is no society for which fertility remains constant over a ten or fifteen year period, it is not very realistic to make such an assumption. Nevertheless, upgrading the fertility rates obtained from the data on births during the year before the census on the basis of the reported census mean parities, at least in some situations, is a better approach than to

merely accept the rates as they are. Moreover, if the changes in level of fertility during the past ten or fifteen years have been modest, then the upgrading provided by the census mean parities may be quite reasonable.

If we assume that fertility and mortality have changed very little during the past ten to fifteen years, the mean parity reported by women aged 25-29 can be used to upgrade the age-specific fertility rates derived from the returns on childbirth during the year before the census. Let us denote these rates by $\hat{f}(x)$. We assume that the estimated rates $\hat{f}(x)$ are proportional to the true, but unknown, rates $f(x)$, i.e.

$$\hat{f}(x) = \kappa f(x) \quad (19)$$

where κ is an unknown constant. Letting \hat{P}_3 denote the observed mean parity for women in the age group [25,30[, we have, at least in an approximate sense, that

$$\hat{P}_3 = \kappa \int_0^{27.5} \hat{f}(x) dx = R \int_0^{27.5} g(x; \hat{c}, \hat{k}) dx$$

where g is given by (12) and where c and k have been estimated from the moments of $\hat{f}(x)$.

It then follows that estimates of R and κ are given by

$$\hat{R} = \hat{P}_3 / \int_0^{27.5} g(x; \hat{c}, \hat{k}) dx \quad (20)$$

and that

$$\hat{\kappa} = \hat{P}_3 / \int_0^{27.5} \hat{f}(x) dx, \quad (21)$$

respectively.

We now illustrate how the assumption (19) can be used for estimation of age-specific fertility rates that agree with the reported mean parity at ages [25,30]. The data on births during the year before the census are given in Table 14. Because the number of women obviously is very small, we have graduated the observed rates by means of a third-degree polynomial. For convenience, we denote the graduated rates in Table 14 (column 5) by $\hat{f}(x)$. Letting

$$T_1 = \sum (x + 2.5)^i \hat{f}(x)$$

the estimated mean value and variance of the graduated rates become

$$\hat{\mu} = T_1/T_0$$

and

$$\hat{\sigma}^2 = T_2/T_0 - (T_1/T_0)^2.$$

The graduated rates (Table 14, col. 5) now give, $\hat{\mu} = 29.5$ and $\hat{\sigma}^2 = 55.0$. In other words, we have a schedule with relatively high mean age and variance. But this is also what we would expect. For estimation of c and k in (12), we have $\hat{c} = \hat{\mu}/\hat{\sigma}^2$ and $\hat{k} = \hat{\mu}^2/\hat{\sigma}^2$ with $\hat{\mu} = 29.5$ and $\hat{\sigma}^2 = 55.0$. This gives $\hat{c} = 0.5364$ and $\hat{k} = 15.8$ whereby

numerical integration yields $\int_{10}^{27.5} g(x; \hat{c}, \hat{k}) = 0.4262$.

If the age-pattern of fertility is well modeled by a gamma density, then women (in the absence of mortality) have produced about 42.62 percent of their children by age 27.5. The mean parity reported by women aged [25,30[was $\hat{P}_3 = 2.2512$. From (20) we obtain that $\hat{R} = 2.2512/0.4262 = 5.2820$. From (21) we obtain $\hat{\kappa} = 5.2820/4.0540 = 1.3029$. Based on this result (and assuming near stationary mortality and fertility), it would appear that there was 30 percent underreporting of births in the census.

If instead of making use of the gamma density to infer the value of the fertility integral up to age 27.5, we make use of $p_{\phi}(x)$, the results are

$$\int_{14.9562}^{27.5} p_{\phi}(x) = 1.69354$$

and

$$\int_{14.9562}^{50.0354} p_{\phi}(x) = 4.01373.$$

This means that women (in the absence of mortality) with that age-pattern of fertility have produced a proportion of $1.69354/4.01373 = 0.42194$ of their fertility at age 27.5. The estimated total fertility rate becomes $\hat{R} = 2.2512/0.42194 = 5.3354$. The constant for upgrading the observed rates (given stationary mortality and fertility) becomes $\hat{\kappa} = 5.3354 / 4.0540 = 1.3161$. These results are in very good agreement with the ones given by the Brass P_i/F_i method which has been illustrated in the main text of the report.

With this method, the crude birth rate in 1987 is estimated as $CBR = 1.3029 \times 300/10,139 = 38.5$ per 1,000 because 300 births were reported and the total population was 10,139. As noted in the main report, we believe this estimate to be too high. It does not agree very well with the population aged 0 enumerated in the census which was 310 children pointing to a crude birth rate of about 31 per 1,000 population. Our final estimate of the crude birth rate, as will be recalled, is 32 per 1,000 population.

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